1.0 **SPECIAL THEORY OF RELATIVITY**
Introduction

In 1905 young Einstein put forward his special theory of relativity, at that time he was a Technical Expert (Third Class) in the Swiss Patent Office, working on physics in his spare time and in what has been termed “splendid isolation” from physicist in the academic community.

Relativity, an aesthetically appealing theory, is about the nature of space and time. It has survived every one of the many searching experimental tests to which it has been subjected to over a century now. Asked about the influence of relativity on the development of physics, one physicist replied, “ Well, relativity is simply *there*”

Relativity has a reputation, among those who have not studied it, as a difficult subject. It is not mathematically complex that stands in the way of understanding; if you can solve a quadratic equation, you are overqualified. The difficulty lies entirely with the fact that relativity forces us to re-examine critically our ideas of space and time.

Our life experiences are restricted in that we have no direct experience with tangible objects moving faster than a tiny fraction of the speed of light. It is no wonder that our ideas of space and time, modelled by this restricted experience, are also restricted. In much the same way a bacterium, spending its life in a fluid environment dominated by viscous forces, knows nothing of gravity.

My advice, be receptive to new ideas and keep an open mind.

Foundation of special theory of Relativity

The idea that a principle of relativity applies to the properties of material of the physical world is very old: it certainly predates Newton and Galileo, but probably not as far back as Aristotle. What the principle of relativity essentially states is the following:

The laws of physics take the same form in all frames of reference moving with constant velocity with respect to one another.

This is a statement that can be given a precise mathematical meaning: the laws of physics are expressed in terms of equations, and the form that these equations take in different reference frames moving with constant velocity with respect to one another can be calculated by the use of transformation equations- the so-called Galilean transformation in the case of Newtonian relativity. The principle of relativity then requires that the transformed equation have exactly the same form in all frames of reference, in other words that the physical laws are the same in all frames of reference.

The principle of relativity was accepted (in somewhat simpler form i.e. with respect to the mechanical behaviour of bodies) by Newton and his successors, even though,

Newton postulated that underlying it all was 'absolute space and time' which defined the state of absolute rest. He introduced the notion in order to cope with the difficulty of specifying with respect to what an accelerated object is being accelerated. To see what is being implied here, imagine space completely empty of all matter except for two masses joined by a spring. Now suppose that the arrangement is rotated, that is, they undergo acceleration. Naively, in accordance with our experience, we would expect that the masses would pull apart. But why should they? How do the masses 'know' that they are being rotated? There is no 'signposts' in an otherwise empty universe that would indicate that rotation is taking place. By proposing that there existed an absolute space, Newton was able to claim that the masses are being accelerated with respect to this absolute space, Newton was able to claim the masses are being accelerated with respect to this absolute space, and hence that they would separate in the way expected for masses in circular motion. But this was a superposition made more for the convenience it offered in putting together his laws of motion, than anything else. It was an assumption that could not be substantiated, as Newton was well aware- he certainly felt misgivings about the concept. Other scientists were more accepting of the idea, however, with Maxwell's theory of electromagnetism for a time seeming to provide some sort of confirmation of the concept.

Another is thought experiment is, when two observers in two different inertia frames of reference are made to observe an event each will record the same time interval and the same spatial interval. For example dropping a ball from a height, the time interval between the two event; timing and releasing of the ball, the time interval as measured by two observers is the same.

Frame of Reference

Newton's law are of course, the laws, which determined how matter moves through space as a function of time. So in order to give these laws a precise meaning we have to specify how we measure the position of material object, a particle say, and the time at which it is at position. We do this by introducing the notion of frame of reference.

A frame of reference is a set of three mutually perpendicular coordinate axes relative to which the position of a particle in space can be described or its velocity and acceleration can be determined, as well as a clock fixed in this system serve to indicate the time.

Inertial Frames of Reverence and Newton's Law of Motion

There exist frames of reference relative to which a particle acted on by no forces moves in a straight line at a constant speed.
A frame of reference, which has this property, is called an inertia frame of reference, or just an inertial frame. That is the law of Inertia holds in an inertia frame of frame or inertial system. An observer at rest(with zero velocity) in such a system is an inertial observer

This is a statement of Newton's First law of Motion. A

Newton's First Law states: A body at rest remains at rest, ad a body in motion will continue in motion with steady speed in a straight line as long as no outside force acts on the body.

The main idea of an inertia observer in an inertia frame is that the observer experience no acceleration (and therefore no net force). If S is an inertia frame and S' is a frame (i.e. coordinates system) moving uniformly relatively to S, then S' is itself an inertial frame. Frames S and S' are equivalent in the sense that there is no mechanical experiment that can be conducted to determine whether either frame is at rest or in uniform motion (that is, there is no preferred frame) This is called the *Galilean (or classical Principle of Relativity*

A test for an inertial frame is simplicity itself. A frame of reference maybe thought of as a mesh of lines. In free space, in the absence of gravitational or other force fields, a particle set down in an inertial frame may always be found there. If the particle is set in motion, it will move with steady speed in a straight line. Its coordinates, measured in this frame, will satisfy the equation of a straight line, and its motion along this line will be with constant speed. If the particle does not remain at the point at which it is place, or does not move in a straight line with steady speed, then the frame is not inertial.

Experiment has shown that Newton's frame of reference, fixed in the stars, is an inertial frame. A coordinate frame fixed in the earth is not inertia, due to the earth's rotation about its axis, and about the sun as well.
For most practical purpose, and to the accuracy with which many experiments are performed, the fact that a frame fixed in the earth is not inertial does not alter the outcome of experiments sufficiently to cause concern. In fact it was only through the *foucault pendulum* that the rotation of the earth was first clearly demonstrated, in 1851.

CLASSICAL PRINCIPLE OF RELATIVITY:

The theory of relativity is concerned with the way in which observers who are in a state of relative motion describe physical phenomena. The idea of an absolute state of rest or motion has long been abandoned, since an observer "at rest" in an earth-bound laboratory is sharing the motion of the earth about its axis, the earth about the sun, the solar system through the Milky Way, and so on. It is common knowledge that one can perform simple experiments with bouncing balls, oscillating springs, or swinging pendula in a laboratory fixed on the earth or in a smoothly running truck moving at constant velocity and obtain identical results in both sets of experiments (NB. This is only true for experiments that are insensitive to the rotation of the earth). We describe this fact by saying that the laws of motion are *covariant*, that is they retain the same form when expressed in the coordinates of either frame of reference.

Galilean transformation

Through the study of projectile motion, Galileo concluded that the motion of a projectile launched from the ground at an arbitrary angle could be derived from the motion of a projectile launched straight up. A projectile thrown straight up from a uniformly moving cart would be seen from the cart as moving straight up and down, but the motion seen from the ground could be predicted by superimposing the motion of the cart onto that of the projectile. These ideas of Galileo are the basis of the *Galilean transformation,* through which we relate motion as they are observed in two different inertial frames. We will see that Galilean transformations must be superseded by a different transformation, the Lorentz transformation, when the two frames move at great speed with respect to each other. But for the ordinary experience the Galilean transformation is much simpler and it quite satisfactory.

1.1 Events and Coordinates

An *event* is something that happens to which an observer can assign three space coordinates and one time coordinate.

Among many possible events are: (1) the turning *on or off* of a tiny bulb, (2) the collision of two particle, (3) the passage of a pulse of light through a specified point in space, or (4) the coincidence of the hand of a clock with a marker on the rim of the clock.

An observer, fixed in an inertial reference frame, may assign to event A (the turning on of a light bulb, say) the following spacetime coordinates below may apply:

Record of Event A

Coordinate	Value
x	3.58 m
y	-1.29 m
z	2.77 m
t	34.5 s

A given event may be recorded by any number of observers, each in their own inertia reference frame.

1.2 Galilean Coordinate Transformations

Consider two frames having relative translational velocity v in their common x-direction.

Fig. 1 shows two observers O and O′. Observer O′ moves with a constant velocity v relative to O along the x′ axis. Let us assume that the observers have adjustable clocks which are adjusted such that they pass each other at a time $t = t' = 0$ when $x = x' = 0$. For any event such as P, observer O will record the event as (x, y, z, t) while observer O′ records it as (x', y', z', t'). How are these sets of numbers related? It is evident that the coordinates are related somehow.

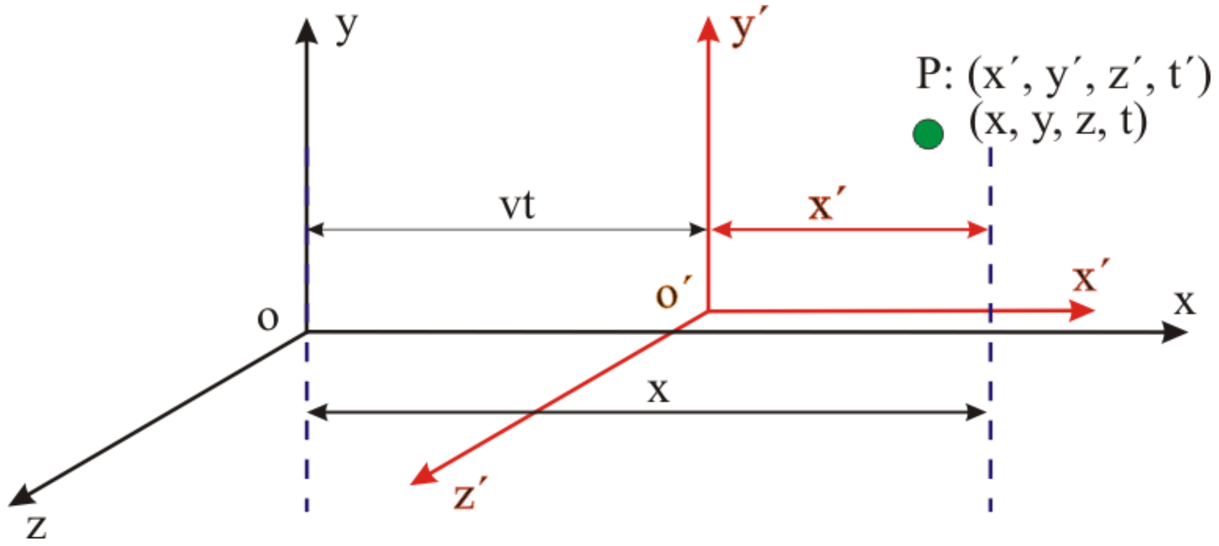

Fig.1

Now for any particular event P the relationship between the coordinates (x, y, z, t) and (x′, y′, z′, t′) for observers O and O′are as follows:

$$x' = (x - vt) \qquad 1.1$$
$$y' = y \qquad 1.2$$
$$z' = z \qquad 1.3$$
$$t' = t \qquad 1.4$$

Equations 1.1 – 1.4 are called the inverse Galilean Coordinate Transformations

The direct transformations are

$$x = (x' + vt') \qquad 1.5$$
$$y = y' \qquad 1.6$$
$$z' = z \qquad 1.7$$
$$t' = t \qquad 1.8$$

1.3 Galilean Velocity Transformations

Let the velocity component measured by observers O and O′ be (u_x, u_y, u_z) and (u_x', u_y', u_z') respectively, then, from equations 1.1 – 1.4, and using the function of a function relation,

$$\frac{dx'}{dt'} = \frac{dx'}{dt} \cdot \frac{dt}{dt'}$$

we have

$$u_x' = \frac{dx'}{dt'} = \frac{d}{dt}(x - vt) \cdot \frac{dt}{dt'} = \frac{dx}{dt} - v \qquad 1.9$$

but $\frac{dx}{dt} = u_x \Rightarrow$ (1.9) becomes,

$$u_x' = u_x - v \qquad 1.10$$

In addition

$$u_y' = u_y \qquad 1.11$$

$$u_z' = u_z \qquad 1.12$$

1.4 Galilean Acceleration Transformations

The acceleration transformations are obtained by either differentiating the distance relation twice or differentiating the velocity relation once. Thus if we let a_x denote the acceleration in the x direction then,

$$a_x = \frac{d^2x}{dt^2} \qquad 1.13$$

or

$$a_x = \frac{du_x}{dt} \qquad 1.14$$

we obtain

$$a_x' = a_x, \quad a_y' = a_y \text{ and } a_z' = a_z \qquad 1.1$$

Thus, it is seen from (1.15) that the acceleration remains constant in all observers moving with a uniform velocity v.

$$F' = ma' = ma = F \qquad 1.11$$

Equation 1.11 says that Newton's laws have the same form in both coordinate frames that is they are covariant. Note that the coordinates and velocities are different in the two frames but each observer always knows how to obtain the other's value. Frame of references in which Newton's laws are covariant are called inertial frames. Inertial frames are equivalent in the sense that there is no mechanical experiment, which can distinguish whether either frame is at rest or in uniform motion; hence,

there is no preferred frame. This is known as the Galilean or classical principle of relativity and the coordinate transformation given above is a Galilean transformation. Strictly speaking, the earth is not an inertial frame because of its rotation and is orbital motion, but it can often be treated as an inertial frame without serious error.

Theory of special relativity

Although Newtonian mechanics gives an excellent description of Nature, it is not universally valid. When we reach extreme conditions—the very small, very heavy or the very fast—the Newtonian Universe that were used needs replacing. You could say that Newtonian mechanics encapsulates our common sense view of the world. One of the major themes of twentieth century physics is that when you look away from our everyday world, common sense is not much.

One such extreme is when particles travels very fast. The theory that replaces Newtonian mechanics is due to Einstein. It is called special theory of relativity. The effects of special relativity become apparent only when the speeds of particles become comparable to the speed of light in the vacuum. The speed of light is

$$c = 299\ 792\ 458 \text{ m/s}$$

The value of is c exact base on our choice of the unit of length.

1.5 Postulates of the Special Theory of Relativity

The two postulates are due to Einstein are:

1. *The Relativity Principle*
   The laws of physics have the same form in all inertial reference frames. No frame is singled out as preferred.

2. *Constancy of the Speed of Light*
   The speed of light in free space (empty space) has the same value ***c*** in all directions and in all inertial reference frames.

   Or we can state it as:

Light propagates through empty space with a definite speed **c** independent of the speed of the source or observer.

1.5.1 Simultaneous event

If two observers are in relative motion, they will not, in general, agree as to whether two events are simultaneous. If one observer find them to be simultaneous, the other will not, and conversely.
We cannot say that one observer is right and the other wrong. The situation is completely symmetrical and there is no reason to choose one observer over the other.

We conclude the following. Simultaneity is not an absolute concept but a relative one, depending on the state of motion of the observer.

1.6 Inconsistency of the Galilean Transformation with Einstein's Postulates (The Lorentz Transformation)

1. If light moves along the x-axis with a speed $u'_x = c$, then the Galilean transformation equations imply that the speed in the x′ frame would be $u_x = c + v$ instead of the expected value of $u_x = c$ which is consistent with Einstein's postulates and also with experiment.
2. The classical transformation equation therefore will have to be modified to make consistent with Einstein's postulates.
3. This is done by introducing a constant γ, which is independent of the coordinates in the modified equation. This gives the equations the form

$$x = \gamma(x' + vt') \qquad 1.16$$

γ depends on v or c.

Likewise,

$$x' = \gamma(x - vt) \qquad 1.17$$

1.6.1 Thought Experiment

Let us consider a propagating light source measured by two different observers from the x and x′ coordinate systems.

Assumptions: $t - t' = 0$, $t' = 0$

For the x-coordinate system, the equation for the propagating wave front is given by

$$x = ct \tag{1.18}$$

Likewise for the x′- coordinate system,

$$x' = ct' \tag{1.19}$$

Substituting 1.18 and 1.19 in 1.16 and 1.17 gives

$$ct = \gamma(ct' + vt') = \gamma(c + v)t' \tag{1.20}$$

and

$$ct' = \gamma(ct + vt) = \gamma(c - v)t \tag{1.21}$$

Eliminating $\frac{t'}{t}$ from 1.20 and 1.21 and determining γ gives

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.22}$$

Since $v < c$, $\gamma > 1$. For $v << c$, $\gamma \approx 1$.

Substituting $x = \gamma(x' + vt')$ from 1.16 into 1.17 gives

$$x' = \gamma[\gamma(x' + vt') - vt] \tag{1.23}$$

Solving 1.23 for t in terms of t' and x' gives

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \tag{1.24}$$

The complete transformations (i.e. Lorentz Transformations) are thus:

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \tag{1.25}$$

The corresponding inverse transformations are

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \tag{1.26}$$

1.7 Time Dilation

Let us consider <u>two events</u> that occur at times t_1' and t_2' in the frame x' at a common point x_0'. The corresponding times recorded for the <u>same event</u> in the x frame will be t_1 and t_2.

From equations 1.24 and 1.25

$$t_1 = \gamma\left(t_1' + \frac{vx_0'}{c^2}\right) \tag{1.27}$$

and

$$t_2 = \gamma\left(t_2' + \frac{vx_0'}{c^2}\right) \tag{1.28}$$

$$\Rightarrow \qquad t_2 - t_1 = \gamma(t_2' - t_1') \tag{1.29}$$

Note:

1. The time interval between two events that take place at the same place in a reference pegged in this place is called the <u>proper time</u>. This means that the time interval $t_2' - t_1'$ measured in the x' frame is the proper time.
2. The time interval measured in any other reference frame is always found to be <u>longer than the proper time</u>.
3. This time increase is called the time dilation.

From equation 1.29, the time dilation Δt is given by given by

$$\Delta t = \gamma(\Delta t_0)$$

1.30

1.8 Length Contraction

1. The length of an object measured in a reference frame in which the object is at rest is called the proper length L_0.
2. The length L measured in a reference in which the body is moving is always shorter than L_0.
3. The length decrease between L_0 and L is called the length contraction.

Now consider a rod at rest in a frame x' with one end at x_2' and the other at x_1'. The proper length of the rod is thus $L_0 = x_2' - x_1'$. In the frame x the rod moves with a speed v with a length $L = x_2 - x_1$.
Note:

The length of the rod is observed at the same time $t_1 = t_2$.

From equation 1.17,

$$x_2' = \gamma(x_2 - vt_2) \text{ and } x_1' = \gamma(x_1 - vt_1)$$

and

$$x_2' - x_1' = \gamma(x_2 - x_1)$$

1.31

using $t_1 = t_2$

Equation 1.31 gives

$$x_2 - x_1 = \frac{1}{\gamma}(x_2' - x_1')$$

1.32

Or

$$L = \frac{1}{\gamma} L_0$$

1.33

1.9 Worked Examples

1. A spaceship passes you at a speed of 0.750c. You measure its length to be 28.2m. How long would it be when at rest?

Solution

We measure the contracted length and find the rest length from

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}$$

$$28.2\,\text{m} = L_0\left[1-(0.750)^2\right]^{\frac{1}{2}}$$

which gives L_0 = 42.6m.

2. A certain type of elementary particle 2.70 x 10^8 m/s. At this speed, the average lifetime is measured to be 4.76 x 10^{-6} s. What is the particle's lifetime at rest?

Solution

$$\Delta t = \frac{\Delta t_0}{\left[1-\left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}};$$

$$4.76\times10^{-6}\,\text{s} = \frac{\Delta t_0}{\left\{1-\left[\frac{\left(2.70\times10^8\,\text{m/s}\right)}{\left(3.00\times10^8\,\text{m/s}\right)}\right]^2\right\}^{\frac{1}{2}}}, \text{ which gives } \Delta t_0 = \boxed{2.07\times10^{-6}\,\text{s.}}$$